

# Simple Iterative Approach to Calculate Wet-Bulb Temperature for Estimating Evaporative Cooling Efficiency

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**Abstract** – This paper presents a detailed iterative approach to the calculation of wet-bulb temperature from dry-bulb temperature and relative humidity at various atmospheric pressures. Such procedure is very useful in different applications including the estimation of evaporative cooling efficiency. This approach can substitute the available empirical regression methods that have validity only over a certain range of input values. The presented approach uses a simple iterative solution that can be run using common software packages such as MS-Excel®. Validation of wet-bulb temperatures from the iterative approach against wet-bulb temperatures provided by the equation of state showed that the iterative approach offers very accurate calculation of the wet-bulb temperatures from dry-bulb temperature and relative humidity at different atmospheric pressures. The maximum mean predictive error, mean absolute predictive error and root mean square error (RMSE) did not exceed  $-0.023^{\circ}\text{C}$ ,  $0.025^{\circ}\text{C}$  (standard deviation of  $0.031^{\circ}\text{C}$ ,  $n=120$ ) and  $0.039^{\circ}\text{C}$ , respectively. Over the range of  $-30$  to  $80^{\circ}\text{C}$  dry-bulb temperatures and  $0$  to  $80\%$  relative humidity, the presented approach converged to a final value of wet-bulb temperature in a maximum of only 4 iterations.

**Keywords** – Wet-bulb Temperature, Evaporative Cooling, Efficiency, Psychrometric Calculations.

## I. INTRODUCTION

Wet-bulb temperature is the minimum temperature the air can reach under adiabatic evaporative-cooling process. This temperature is an important meteorological parameter used in the calculation of several applications such as: The Discomfort Index (TDI) [1]–[4], Effective Temperature Index (ETI) [5], [6], Wet-bulb Globe Temperature (WGT) [7]–[9], Temperature-Humidity Index (THI) [10], Tropical Summer Index (TSI) [11] and Fighter Index of Thermal Stress (FITS) [12], [13].

Wet-bulb temperature is also a parameter used in the calculation of humidity ratio once relative humidity ( $RH$ ) and dry-bulb temperature ( $T_{db}$ ) are known [14]. In agricultural practices, wet-bulb temperature is very essential in grain storage applications [15]. It is also involved in calculating the efficiency of evaporative cooling systems which are normally used in summer cooling of commercial greenhouses [16]. Equation (1) depicts the calculation of the evaporative cooling efficiency ( $\eta$ , %) from the inlet air dry-bulb ( $T_{db,in}$ ,  $^{\circ}\text{C}$ ) and wet-bulb ( $T_{wb}$ ,  $^{\circ}\text{C}$ ) temperatures and the outlet air dry-bulb temperature ( $T_{db,out}$ ,  $^{\circ}\text{C}$ ) [17]–[20].

$$\eta = 100 \frac{T_{db,in} - T_{db,out}}{T_{db,in} - T_{wb}} \quad (1)$$

Wet-bulb temperature ( $T_{wb}$ ) is normally measured using psychrometers, hygrometers or other sensing mechanisms

[21]–[23]. However, there is no straightforward analytical approach to calculate  $T_{wb}$  from  $T_{db}$  and  $RH$ . Consequently, this imposes some difficulties for an agricultural engineer attempting to use  $T_{wb}$  for calculating evaporative cooling efficiency, humidity ratio or for any other application while not having proper measuring devices or when  $T_{wb}$  is not provided by weather stations or meteorological forecasts. Reference [24] tried to resolve this predicament by developing an empirical relation calculating  $T_{wb}$  from  $T_{db}$  and  $RH$  at sea-level atmospheric pressure ( $P_{atm} = 101.325$  kPa). This relation was found to have a good accuracy but because it was developed via an empirical regression approach, its applicability is only valid for the same atmospheric pressure over the range of  $T_{db}$  and  $RH$  values involved in the regression (i.e.  $0$  to  $50^{\circ}\text{C}$  and  $1$  to  $99\%$ , respectively). Therefore, it is necessary to repeat the regression analysis for anyone interested in  $T_{wb}$  values outside these conditions.

Reference [25] briefly described an iterative analytical approach for calculating  $T_{wb}$  from  $T_{db}$  and  $RH$  at any  $P_{atm}$ . However, the accuracy of this approach has not yet been investigated. This paper provides detailed procedure on how to use this approach and thoroughly investigates its accuracy.

## II. ITERATIVE APPROACH FOR WET-BULB TEMPERATURE

From [14], it was found that humidity ratio of moist air ( $W$ ) can be estimated from (2) or (3):

$$W = 0.62198 \frac{P_{wv}}{P_{atm} - P_{wv}} \quad (2)$$

$$W = \frac{(L_{v,0} - (C_{p,w} - C_{p,wv}) T_{wb}) W_{s,wb} - C_{p,da} (T_{db} - T_{wb})}{L_{v,0} + C_{p,wv} T_{db} - C_{p,w} T_{wb}} \quad (3)$$

where

$W$  = humidity ratio, kg/kg

$P_{wv}$  = water-vapour partial pressure, kPa

$P_{atm}$  = atmospheric pressure, kPa

$L_{v,0}$  = latent heat of vaporization at  $0^{\circ}\text{C}$ , 2501 kJ/kg

$C_{p,w}$  = specific heat of liquid water, 4.186 kJ/kg $^{\circ}\text{C}$

$C_{p,wv}$  = specific heat of water vapour, 1.805 kJ/kg $^{\circ}\text{C}$

$C_{p,da}$  = specific heat capacity of dry air, 1.006 kJ/kg $^{\circ}\text{C}$

$T_{wb}$  = wet-bulb temperature,  $^{\circ}\text{C}$

$T_{db}$  = dry-bulb temperature,  $^{\circ}\text{C}$

$W_{s,wb}$  = saturation humidity ratio at  $T_{wb}$ , kg/kg

Water-vapour partial pressure,  $P_{wv}$ , in (2) is calculated as a function of  $RH$  and saturation water-vapour partial pressure,  $P_{s,wv}$ , using the following relations [14], [26]:

$$P_{wv} = \frac{RH}{100} P_{s,wv} \quad (4)$$

$$P_{s,wv} = 2.1718 \times 10^7 \exp\left(\frac{-4157}{T_{db} + 239.24}\right) \quad (5)$$

Saturation humidity ratio at wet-bulb temperature,  $W_{s,wv}$ , in (3) is calculated using (2), (4), and (5) but with  $T_{wb}$  replacing  $T_{db}$  in (5). For improved accuracy, it is recommended to calculate specific heat capacities of water, water vapour and dry air ( $C_{p,w}$ ,  $C_{p,wv}$  and  $C_{p,da}$ , respectively) using temperature-dependent equations instead of using constant values because those variables are temperature-dependent. Therefore, the following relations will be used to estimate  $C_{p,w}$ ,  $C_{p,wv}$  and  $C_{p,da}$  [27], [28]:

$$C_{p,w} = 0.0265T_{wb}^2 - 1.7688T_{wb} + 4205.6 \quad (6)$$

$$C_{p,wv} = 0.0016T_{db}^2 + 0.1546T_{db} + 1858.7 \quad (7)$$

$$C_{p,da} = 0.0667[(T_{db} + T_{wb})/2] + 1005 \quad (8)$$

Equation (2) provides  $W$  from the usually-available weather parameters;  $T_{db}$ ,  $RH$  and  $P_{atm}$ . Therefore,  $W$  value obtained from (2) will be used as a reference for  $W$  value obtained from (3). Furthermore,  $T_{wb}$  involved in (3) will be altered until both  $W$  values equal each other. The flow diagram of the iterative solution is clearly illustrated in Fig. 1. Following the procedure presented in the diagram, the iterative solution could be executed using any numerical software. However, because MS-Excel® software is widely used, detailed solution steps using the Solver tool are provided in Fig.A1 of the Appendix.

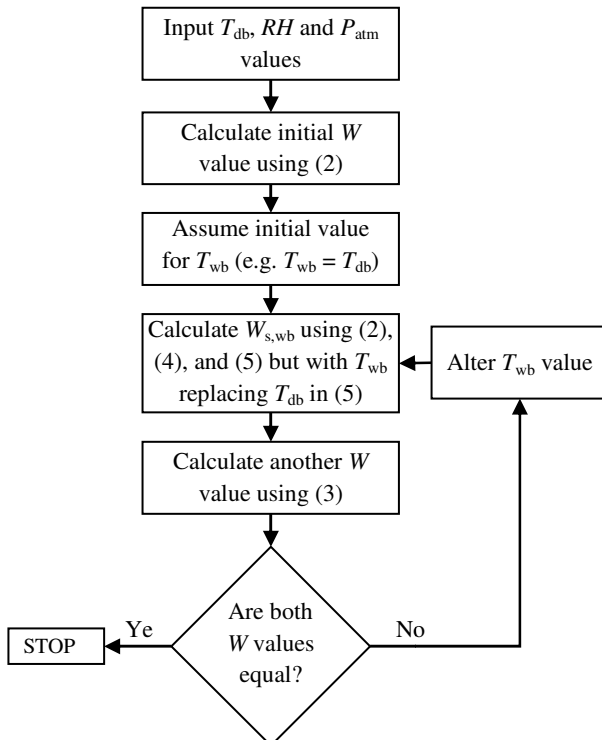


Fig. 1. Flow diagram of the iterative solution for  $T_{wb}$ .

### III. VALIDATION AGAINST REFERENCE DATA

Data generated from the iterative approach were validated against reference data obtained from the thermodynamic calculations of air-water ideal mixture [29]. The reference data of  $T_{wb}$  as a function of  $T_{db}$  and  $RH$  at sea-level atmospheric pressure ( $P_{atm} = 101.325$  kPa) are plotted in Fig. 2. In addition to the graphical illustrations, four statistical parameters are used to study the accuracy of the iterative solution presented in this article. These parameters are the mean predictive error (PE, °C), mean absolute predictive error (APE, °C), root mean square error (RMSE, °C) and coefficient of variation (CV, %).

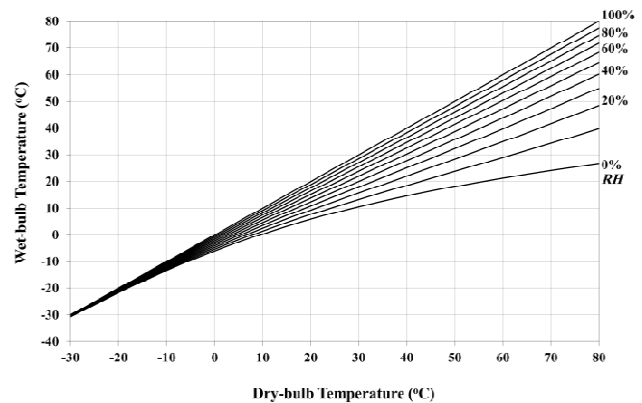


Fig. 2. Equation-of-state reference wet-bulb temperature ( $T_{wb}$ ) as influenced by dry-bulb temperature ( $T_{db}$ ) and relative humidity ( $RH$ ) at sea-level ( $P_{atm} = 101.325$  kPa).

### IV. RESULTS AND DISCUSSION

The accuracy of the iterative approach presented in this study was evaluated using the reference data illustrated in Fig. 2 at  $P_{atm} = 101.325$  kPa as well as the reference data for different  $P_{atm}$ . This evaluation is presented henceforward.

Figs. 3, 4, and 5, respectively, compare the  $T_{wb}$  values estimated using the iterative approach to the values obtained from the equation-of-state at three dry-bulb temperature ranges; low (-30 to 0°C), normal (0 to 50°C) and high (50 to 80°C) for  $P_{atm} = 101.325$  kPa. These temperature ranges were specified according to ASHRAE's terminology [14]. It is clearly seen from these charts that the calculated  $T_{wb}$  is almost equal to the equation-of-state values ( $RMSE \leq 0.04^\circ C$ ) and the variation between them slightly increases as  $T_{db}$  increases. The largest variation (mean error =  $-0.12^\circ C$ ) happened at the high range of dry-bulb temperatures (50-80°C) which is still a very minor variation. Because most agricultural applications are within the normal range of dry-bulb temperatures (0-50°C) then the analytical approach stays valid over a wide range of air temperatures.

Similar to the accuracy at sea-level atmospheric pressure, the analytical approach accurately calculated  $T_{wb}$  at higher altitudes. Figs. 6 and 7 illustrate calculated  $T_{wb}$  against the equation-of-state values over the normal range of dry-bulb temperatures at  $P_{atm}$  of 84.56 and 77.04 kPa, respectively. Although the model accurately predicted  $T_{wb}$  for low and high ranges of dry-bulb temperatures, the comparison charts are not provided for these particular

ranges since most agricultural applications use the normal range of temperatures. Though, Table I provides statistical details about the accuracy of the analytical approach in terms of variations from the equation-of-state values (i.e. Errors) for all ranges of dry-bulb temperature.

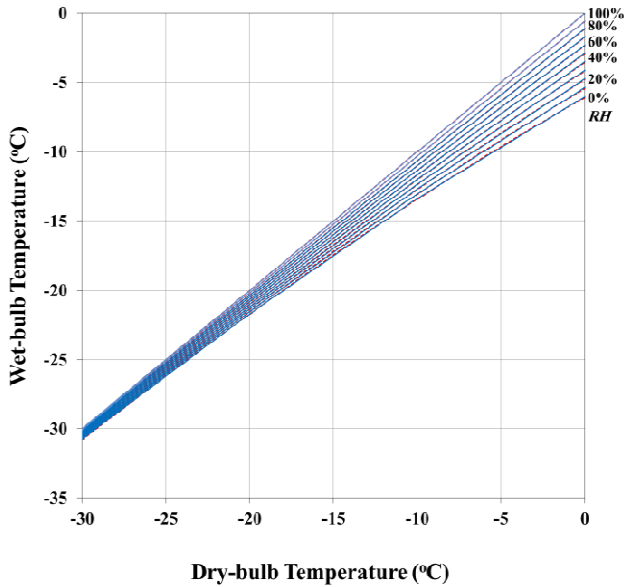


Fig. 3. Reference (solid lines) and calculated (dotted lines) wet-bulb temperature at sea-level ( $P_{atm} = 101.325$  kPa) and low temperature range (-30 to 0°C) for different relative humidity ( $RH$ ) values.

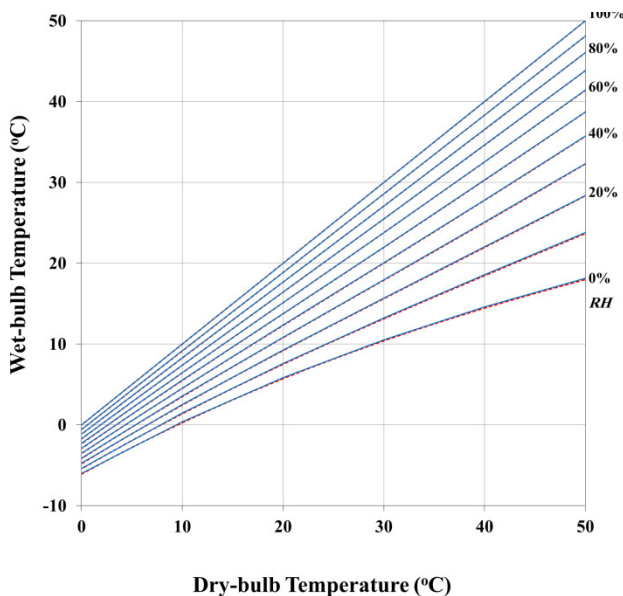


Fig. 4. Reference (solid lines) and calculated (dotted lines) wet-bulb temperature at sea-level ( $P_{atm} = 101.325$  kPa) and normal temperature range (0 to 50°C) for different relative humidity ( $RH$ ) values.

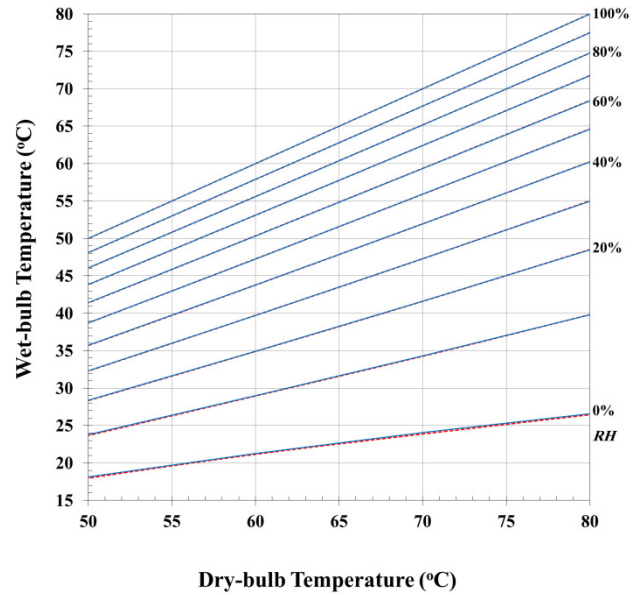


Fig. 5. Reference (solid lines) and calculated (dotted lines) wet-bulb temperature at sea-level ( $P_{atm} = 101.325$  kPa) and high temperature range (50 to 80°C) for different relative humidity ( $RH$ ) values.

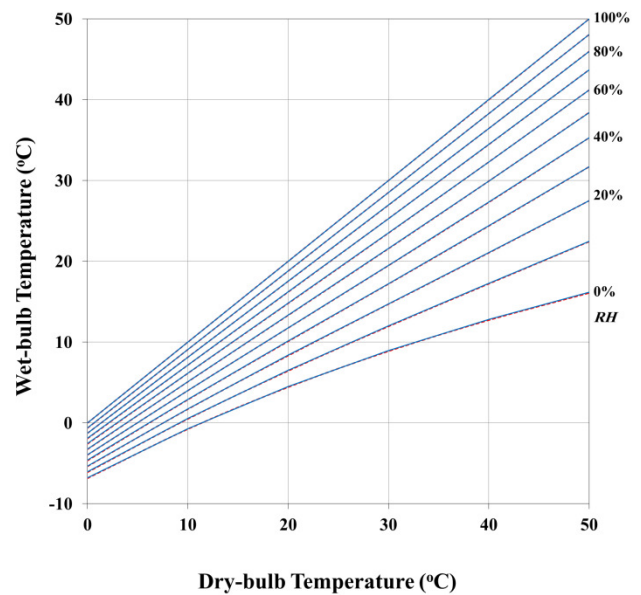


Fig. 6. Reference (solid lines) and calculated (dotted lines) wet-bulb temperature at 1500 m altitude ( $P_{atm} = 84.56$  kPa) and normal temperature range (0 to 50°C) for different relative humidity ( $RH$ ) values.

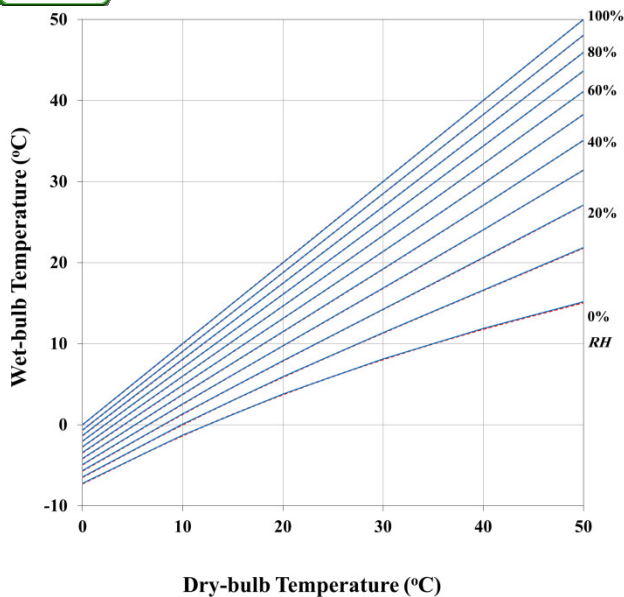


Fig. 7. Reference (solid lines) and calculated (dotted lines) wet-bulb temperature at 2250 m altitude ( $P_{atm} = 77.04$  kPa) and normal temperature range (0 to 50°C) for different relative humidity ( $RH$ ) values.

When we compare the iterative approach presented in this study with the empirical method in[24], we can observe that the iterative approach has a wider range of input values ( $T_{db}$ : -30 to 80°C and  $RH$ : 0 to 100%) as compared to that in[24] ( $T_{db}$ : 0 to 50°C and  $RH$ : 1 to 99%). We also found that the accuracy of the iterative approach (PE: -0.001 to -0.023°C and APE < 0.026°C) is better than the empirical method (PE: +0.65 to -1.00°C and APE

<0.30°C). Although the approach presented in this paper requires an iterative procedure, it was observed that the solution was reached in a maximum of 4 iterations over the range of -30 to 80°C dry-bulb temperatures and 0 to 80% relative humidity. The iterative solution presented in this paper converges to a final answer in a remarkably lower number of iterations than other available algorithms such as the one presented in the following web link: <https://www.easycalculation.com/weather/learn-dewpoint-wetbulb.php>. The latter needs almost 17 iterations to reach to a final answer which is definitely going to increase the calculation time and processing capacity.

## V. CONCLUSIONS

The iterative approach presented in this paper is very simple and very accurate at the same time. Also, it can be solved using any numerical software such as MS-Excel®. Unlike empirical regression methods, this iterative approach offers the possibility to use the atmospheric pressure at elevations other than mean sea level without the need to readjust equations. This approach accurately calculated  $T_{wb}$  from  $T_{db}$  and  $RH$  and the maximum mean predictive error, mean absolute predictive error and root mean square error (RMSE) did not exceed -0.023°C, 0.025°C (standard deviation of 0.031°C,  $n=120$ ) and 0.039°C, respectively. In addition to its simplicity and wide applicability, it converges to a final value of  $T_{wb}$  in a maximum of 4 iterations.

Table 1. Statistical details on the accuracy of the analytical approach for  $T_{wb}$  at different  $P_{atm}$

Statistical Indicators	Dry-bulb temperatures								
	Low Range (-30 to 0°C)			Normal Range (0 to 50°C)			High Range (50 to 80°C)		
	101.325 [kPa]	84.560 [kPa]	77.040 [kPa]	101.325 [kPa]	84.560 [kPa]	77.040 [kPa]	101.325 [kPa]	84.560 [kPa]	77.040 [kPa]
<sup>1</sup> PE, °C	-0.013 <sup>2</sup> (0.011)	-0.016 (0.012)	-0.016 (0.012)	-0.023 (0.025)	-0.023 (0.026)	-0.022 (0.027) <sup>4</sup>	-0.006 (0.039)	-0.003 (0.039)	-0.001 (0.039)
<sup>3</sup> APE, °C	0.013 (0.011)	0.016 (0.012)	0.016 (0.012)	0.024 (0.025)	0.024 (0.025)	0.024 (0.025)	0.023 (0.031)	0.024 (0.031)	0.025 (0.030)
<sup>4</sup> RMSE, °C	0.017	0.020	0.020	0.034	0.035	0.034	0.039	0.039	0.039
<sup>5</sup> CV, %	-0.826	-0.774	-0.767	-1.103	-1.155	-1.195	-6.568	-14.472	-43.296

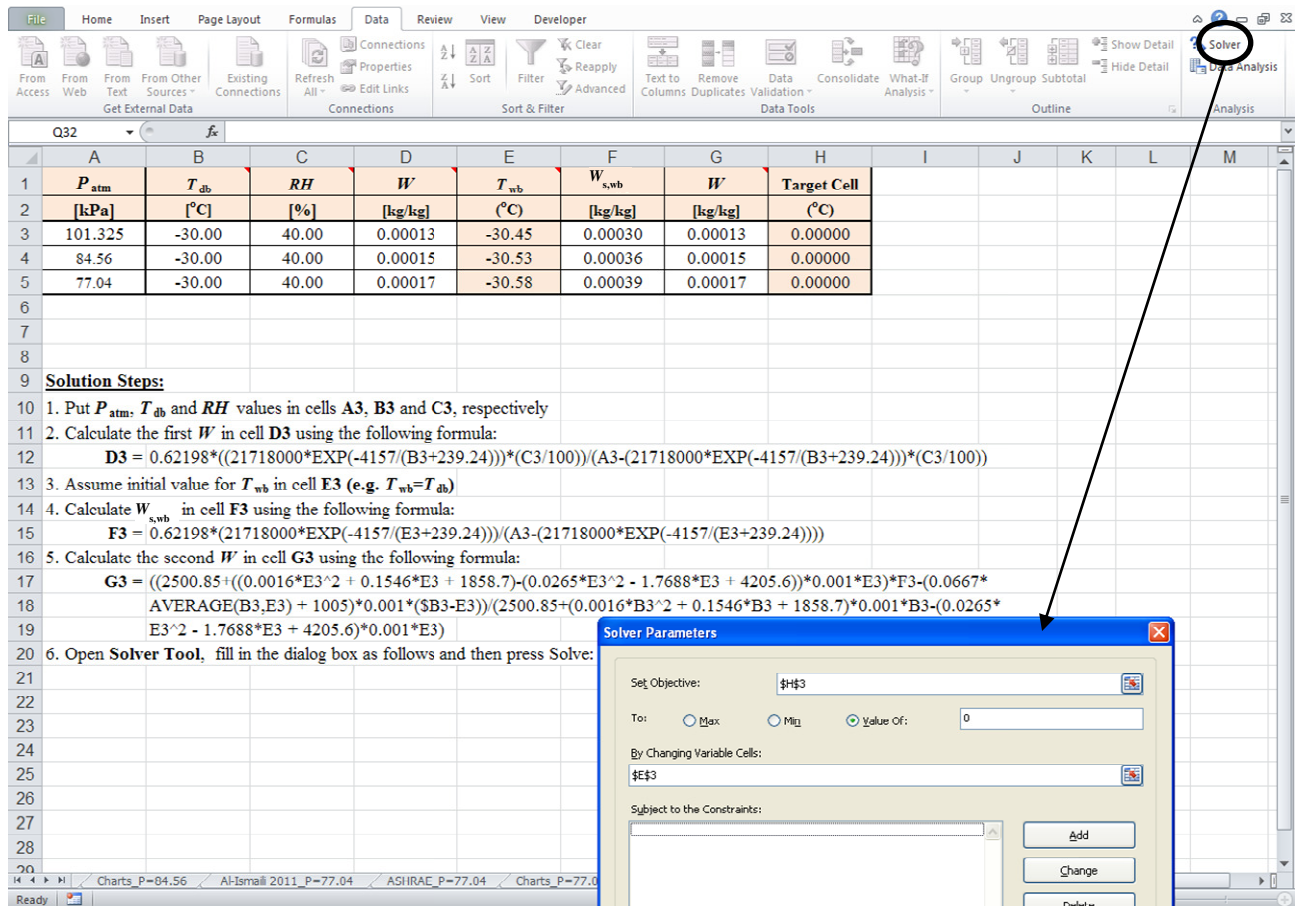
<sup>1</sup> mean predictive error in wet-bulb temperature

<sup>2</sup> numbers between parentheses are for standard deviation

<sup>3</sup> mean absolute predictive error

<sup>4</sup> root mean square error

<sup>5</sup> coefficient of variation

**APPENDIX**


The screenshot shows an Excel spreadsheet with the following data in rows 1-5:

	A	B	C	D	E	F	G	H
1	$P_{atm}$	$T_{db}$	$RH$	$W$	$T_{wb}$	$W_{s,wb}$	$W$	Target Cell
2	[kPa]	[°C]	[%]	[kg/kg]	(°C)	[kg/kg]	[kg/kg]	(°C)
3	101.325	-30.00	40.00	0.00013	-30.45	0.00030	0.00013	0.00000
4	84.56	-30.00	40.00	0.00015	-30.53	0.00036	0.00015	0.00000
5	77.04	-30.00	40.00	0.00017	-30.58	0.00039	0.00017	0.00000

**Solution Steps:**

- Put  $P_{atm}$ ,  $T_{db}$  and  $RH$  values in cells A3, B3 and C3, respectively
- Calculate the first  $W$  in cell D3 using the following formula:  

$$D3 = 0.62198 * ((21718000 * EXP(-4157 / (B3 + 239.24))) * (C3 / 100)) / ((A3 - (21718000 * EXP(-4157 / (B3 + 239.24)))) * (C3 / 100))$$
- Assume initial value for  $T_{wb}$  in cell E3 (e.g.  $T_{wb} = T_{db}$ )
- Calculate  $W_{s,wb}$  in cell F3 using the following formula:  

$$F3 = 0.62198 * ((21718000 * EXP(-4157 / (E3 + 239.24))) / (A3 - (21718000 * EXP(-4157 / (E3 + 239.24))))$$
- Calculate the second  $W$  in cell G3 using the following formula:  

$$G3 = ((2500.85 + ((0.0016 * E3^2 + 0.1546 * E3 + 1858.7) - (0.0265 * E3^2 - 1.7688 * E3 + 4205.6)) * 0.001 * E3) * F3 - (0.0667 * AVERAGE(B3, E3) + 1005) * 0.001 * (B3 - E3)) / (2500.85 + (0.0016 * B3^2 + 0.1546 * B3 + 1858.7) * 0.001 * B3 - (0.0265 * E3^2 - 1.7688 * E3 + 4205.6) * 0.001 * E3)$$
- Open Solver Tool, fill in the dialog box as follows and then press Solve:

The Solver Parameters dialog box is shown with the following settings:

- Set Objective:  $\$H\$3$
- To:  Max  Min  Value Of: 0
- By Changing Variable Cells:  $\$E\$3$
- Subject to the Constraints: (empty)

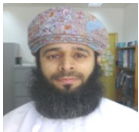
Fig. A1. Solution steps of the analytical approach using MS-Excel®

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